

The Sun and the Newton Constant

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Abstract

Several properties of the solar interior are determined with a very high accuracy, which in some cases is comparable to that achieved in the determination of the Newton constant G_N . We find that the present uncertainty $\Delta G_N/G_N = \pm 1.5 \cdot 10^{-3}$ has significant effects on the profile of density and pressure, however it has negligible influence on the solar properties which can be measured by means of helioseismology and ^8B neutrinos. Our result do not support recent claims that observational solar data can be used to determine the value of G_N with an accuracy of few part in 10^{-4} . Present data cannot constrain G_N to much better than 10^{-2} .

I. INTRODUCTION

In the last ten years, our observational knowledge of the solar interior has progressed enormously. By means of helioseismic data it has become possible to derive the sound speed in the solar interior with accuracy of about one part per thousand [1,2]. By the same method, it has been possible to deduce important properties of the convective envelope. The photospheric Helium fraction Y_{ph} and depth of the convective envelope R_b have been determined with accuracy of about one per cent and one per thousand respectively, following the pioneering papers by [3] and [4]. The measurement of the neutrino flux from Boron decay, obtained by combining SNO and Super-Kamiokande data [5–8], has provided a determination of the temperature T_c near the solar center with accuracy of about one per cent [9].

It is impressive that all the predictions of the Standard Solar Model (SSM) have been confirmed by these accurate tests, see e.g. [1,2] and the first rows of table I for a summary of the available information.

The SSM, like any stellar evolution calculation, depends on several parameters. In this respect the Newton constant G_N plays an important role, since stellar evolution results from the equilibrium between gravitation and other interactions. The sensitivity of stellar evolution to modifications of G_N was first stressed by Teller [10]. By means of a homology argument he demonstrated that the stellar luminosity is $L_\odot \propto G_N^7 M_\odot^5$, so that even a ten

per cent variation of G_N from its standard value would imply that life on Earth cannot be sustained!

Among the “fundamental parameters” of nature [11,12], G_N is the most difficult to measure. In fact, it is determined with relatively poor accuracy and the situation of the field can be summarized by observing that the CODATA-98 [12] value, $G_N = 6.673(10)10^{-11} \text{ m}^3\text{s}^{-2}\text{Kg}^{-1}$ includes a relative accuracy $\Delta G_N/G_N = \pm 1.5 \cdot 10^{-3}$ which is a factor ten larger than that estimated in 1986 [13]. Recent experiments [14–16] have claimed accuracy of about 10^{-5} however the disagreement between individual results is at the level of 10^{-3} .

Since several solar properties are now observationally determined with an accuracy of order 10^{-3} , we shall address the following questions:

- 1) *What is the uncertainty on SSM predictions induced by the uncertainty of G_N ?*
- 2) *Can one exploit the available accurate observations of the solar interior, i.e. helioseismic data and Boron neutrino flux, for obtaining a determination of G_N with accuracy comparable or better than that of laboratory measurements?*

This last question, which is particularly interesting, was recently raised by Lopes and Silk [17].

We shall present solar models calculated for different value of G_N , deriving the predictions to be compared with helioseismic and neutrino data. We shall find that solar sound speeds and temperature are actually very weakly dependent on G_N and we shall provide an explanation of this apparently puzzling result.

II. SOLAR MODELS

We have built several solar models corresponding to different values of G_N , by using an up-to-date version of the evolutionary code FRANEC, which includes element diffusion, recent opacity tables and modern nuclear reaction rates [18].

For a given value of G_N the three input parameters of the code - the mixing length α , the initial Helium and metal abundances Y_{in} and Z_{in} are varied until one reproduces the observed values of the solar radius [$R_\odot = (6.9598(1 \pm 0.01\%)10^8 \text{ m}$], luminosity [$L_\odot = 3.844(1 \pm 0.4\%)10^{26} \text{ W}$] and photospheric composition [$Z/X = 0.0245(1 \pm 6\%)$] at the solar age [$t_\odot = 4.57(1 \pm 0.4\%) \text{ Gy}$].

We remark that the calculated properties of the solar interior are sensitive to the values of these observables, particularly to L_\odot . For this reason, in order to disentangle the effect of tiny changes of G_N we require that L_\odot is fixed to the level of $2 \cdot 10^{-5}$, i.e. much better than the observational accuracy. This precaution is necessary, otherwise the calculated models will reflect the changes of L_\odot mixed to the changes of G_N .

We also remark that astronomical observations fix the product $G_N M_\odot$ quite accurately [19]:

$$G_N M_\odot = (132\,712\,438 \pm 5)10^{12} \text{ m}^3\text{s}^{-2} \quad . \quad (1)$$

Laboratory measurements of G_N are thus measurements of M_\odot . If G_N is changed M_\odot has to be varied so that eq.(1) is satisfied. This is an important point for the present discussion.

In Figs. 1,2,3 and in table I we present the effect of varying G_N by $\pm 1\%$ with respect to the “standard” CODATA-98 value. The following points are to be remarked:

i) the changes of pressure and density are of the same order of $\delta G_N/G_N$, as expected, see Fig. 1.

ii) On the other hand the induced variation of the (squared isothermal) sound speed $u = P/\rho$ is much smaller. For $\delta G_N/G_N = 1\%$ one has at most $\delta u/u = 2 \cdot 10^{-3}$, a value comparable to the present observational accuracy, see Fig. 2.

iii) Also the variation of temperature is much suppressed, see Fig. 3: the change of the central temperature is about 0.3%, and that of the Boron flux is about 5%, see Table I, in both cases a factor three below the present observational uncertainty.

iv) The depth of the convective zone is altered just by a factor $4 \cdot 10^{-4}$ and the photospheric helium abundance by less than 1%, well below the present observational uncertainties, see again Table I.

More generally, we can express the sensitivity of the observable O_i to the change of G_N by using scaling parameters

$$\beta_i = \frac{d \log O_i}{d \log G_N} \quad . \quad (2)$$

The calculated β_i values are also presented in Table I. Changes in G_N at the level of present uncertainty ($1.5 \cdot 10^{-3}$) induce changes of helioseismic observables and neutrino fluxes which are negligible in comparison with the respective observational uncertainty, see last row of Table I.

Our results do not support the claim of ref. [17] that observational solar data can be used to determine the value of G_N with accuracy of few parts in 10^{-4} . Variations of G_N at this level induce changes which are much too small in comparison with observational accuracy.

In summary, present data are sensitive to changes of G_N provided that these are of the order of 10^{-2} .

III. INTERPRETATION

A puzzling situation has emerged. G_N is clearly an important parameter, and its variation induces a comparable change on physically relevant quantities such as P and ρ . On the other hand the change of u and T are definitely suppressed.

The reason for the cancellation is in the constancy of $G_N M_\odot$, as can be easily demonstrated.

Actually the typical scales of solar pressure and density are given by:

$$P = G_N M_\odot^2 / R_\odot^2 \quad (3)$$

$$\rho = M_\odot / R_\odot^3 \quad (4)$$

For a change of G_N which keeps constant the product of $G_N M_\odot$ these vary as :

$$\delta P/P = \delta \rho/\rho = -\delta G_N/G_N. \quad (5)$$

In fact, this is the pattern shown in Fig. 1.

In this approximation, when considering $u = P/\rho$ the effects of changing G_N cancel, in other words u is unaffected by changes of G_N , which explains the much weaker sensitivity emerging from the numerical calculations presented in the previous sections.

From the perfect gas law, which describes to a good approximation most of the solar core, one has $P/\rho = kT/\mu$, so that also T/μ is weakly sensitive to changes of G_N . The present value of the mean molecular weight μ is mainly determined from the initial conditions (Y_{in}) and the solar history and not from G_N . It follows that also T is very weakly sensitive to G_N .

We remark that the constancy of $G_N M_\odot$ is essential for the cancellation. If one computes stellar structures with fixed M_\odot and different G_N one finds fractional changes of u and T that are proportional to $\delta G_N/G_N$, however these cannot be interpreted as solar models.

IV. FUTURE PROSPECTS

One expects that, in the future, the sound speed u will be measured with a higher accuracy, possibly to the level of 10^{-4} . Even in this case, however, it will be difficult to get significant improvements on G_N , unless one achieves the same level of accuracy in several other physical inputs which affect u . For instance, let us consider the effect of variations of the nuclear reaction cross sections and of the opacity, κ . We remind that these quantities are affected by uncertainties of few percent (at least), see [21–23].

As we have already remarked, the product $G_N M_\odot$ is quite accurately fixed, so that if G_N increases, the solar mass must decrease according to $\delta M_\odot/M_\odot = -\delta G_N/G_N$. A smaller M_\odot , being the radius of the sun fixed by observational data, implies a reduction of number densities n_i of the various particles present in the sun.

This implies, a reduction of the nuclear reaction rates (which are proportional to $n_i n_j$) and, at the same time, an increase of the photon mean free path (which, in the core of the sun, is inversely proportional to electron number density). This suggests that suitable changes of nuclear energy production rate ϵ and of opacity can mimic changes of G_N .

Actually, one expects that a combined variation $\delta\epsilon/\epsilon = -2\delta G_N/G_N$ and $\delta\kappa/\kappa = -\delta G_N/G_N$ should affect the sound speed in the same way as a variation of G_N . This is supported by the numerical results shown in Fig. 4. We have built a solar model with energy production rate decreased by 2% and with opacity decreased by 1%, and we have obtained a sound speed profile which is similar (at the level 0.1% or less) to that obtained by increasing G_N by 1%.

Similar considerations hold for a possible determination of G_N by means of Φ_B measurements. By using the β coefficient shown in Tab. I, one sees that a variation $\delta G_N/G_N = 10^{-3}$ corresponds to change of the ^8B neutrino flux $\delta\Phi_B/\Phi_B = 5 \cdot 10^{-3}$. A “global” accuracy (i.e. both theoretical and experimental) of the order $5 \cdot 10^{-3}$ is thus needed in order to distinguish among values of G_N which differ by 10^{-3} . We remind that the present experimental and theoretical determinations of the ^8B neutrino flux have uncertainties of the order of 15-20% per cent. ns hold for a possible determination of G_N by means of Φ_B measurements. By using the β coefficient shown in Tab. I, one sees that a variation $\delta G_N/G_N = 10^{-3}$ corresponds to change of the ^8B neutrino flux $\delta\Phi_B/\Phi_B = 5 \cdot 10^{-3}$. A “global” accuracy (i.e. both theoretical and experimental) of the order $5 \cdot 10^{-3}$ is thus needed in order to distinguish among values of G_N which differ by 10^{-3} . We remind that the present experimental and theoretical determinations of the ^8B neutrino flux have uncertainties of the order of 15-20% per cent.

V. CONCLUDING REMARKS

We have found that:

- The present uncertainty on the gravitational constant, $\Delta G_N/G_N = \pm 1.5 \cdot 10^{-3}$ has significant effects on the profile of density and pressure.
- On the other hand, it has negligible influence on the solar properties which can be measured by means of helioseismology and ^8B neutrinos: sound speed profile, central temperature, depth and helium content of the convective envelope.
- Our result do not support recent claims [17] that observational solar data can be used to determine the value of G_N with an accuracy of few part in 10^{-4} . Present data cannot constrain G_N to much better than 10^{-2} . Furthermore, even if the sound speed measurements will become much more accurate, it will be difficult to get significant improvements on G_N due to the approximate degeneracy with variations of other physical inputs, e.g. nuclear cross sections and opacity, which are presently known at the per cent level or worse.

ACKNOWLEDGMENTS

We are grateful to G. Fiorentini for useful comments and suggestions.

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TABLES

TABLE I. SSM predictions, observational errors and sensitivity to G_N . Δ is the 1σ relative observational error; $\delta_{\pm 1\%}$ is the variation (model-SSM)/model obtained when changing G_N by $\pm 1\%$; β is the scaling coefficient of eq. 2; $\delta_{+0.15\%}$ is the variation (model-SSM)/model estimated when changing G_N by $+0.15\%$.

Observable units	$u(0.1R_\odot)$ (Mm/s) ²	$u(0.6R_\odot)$ (Mm/s) ²	T_c 10 ⁷ K	Φ_B 10 ⁶ cm ⁻² s ⁻¹	Y_{ph}	R_b R_\odot
SSM [20]	0.1525	0.09222	1.568	5.15	0.2437	0.7140
Δ (%)	0.2*	0.12*	1**	18**	1.4*	0.2*
$\delta_{+1\%}$ (%)	+0.18	-0.036	+0.31	+5.7	-0.67	+0.04
$\delta_{-1\%}$ (%)	-0.20	+0.021	-0.28	-4.4	+0.77	-0.04
β	+0.19	-0.03	+0.29	+5.1	-0.72	+0.044
$\delta_{+0.15\%}$ (%)	+0.028	-0.0045	+0.044	+0.76	-0.11	+0.0065

*from helioseismic data [1]

** from ⁸B neutrino data [5,9]

FIGURES

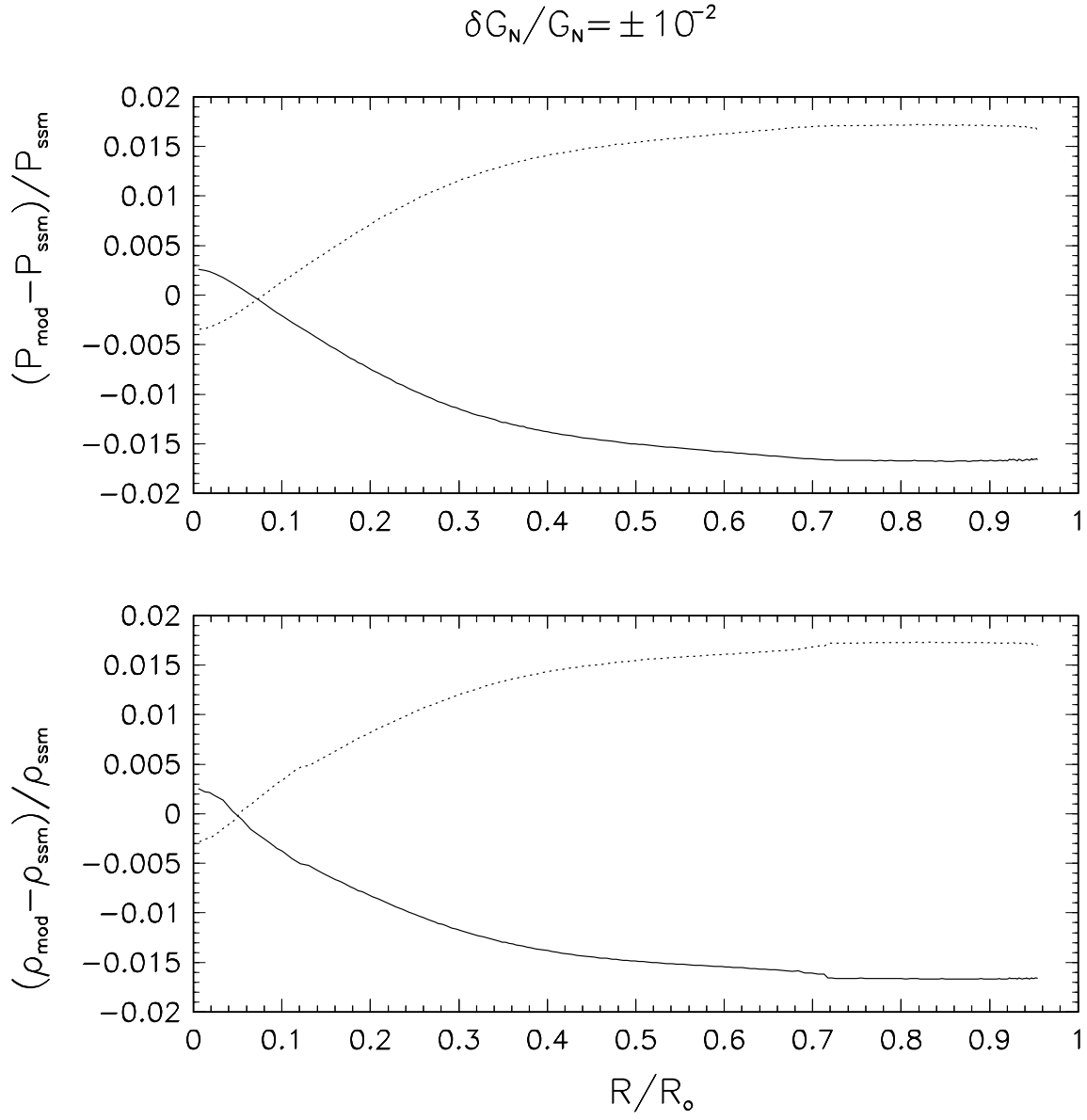


FIG. 1. Relative change (model-SSM)/SSM of pressure (upper panel) and density (lower panel) as a function of the radial coordinate for a change of G_N by +1% (full line) and -1% (dashed line).

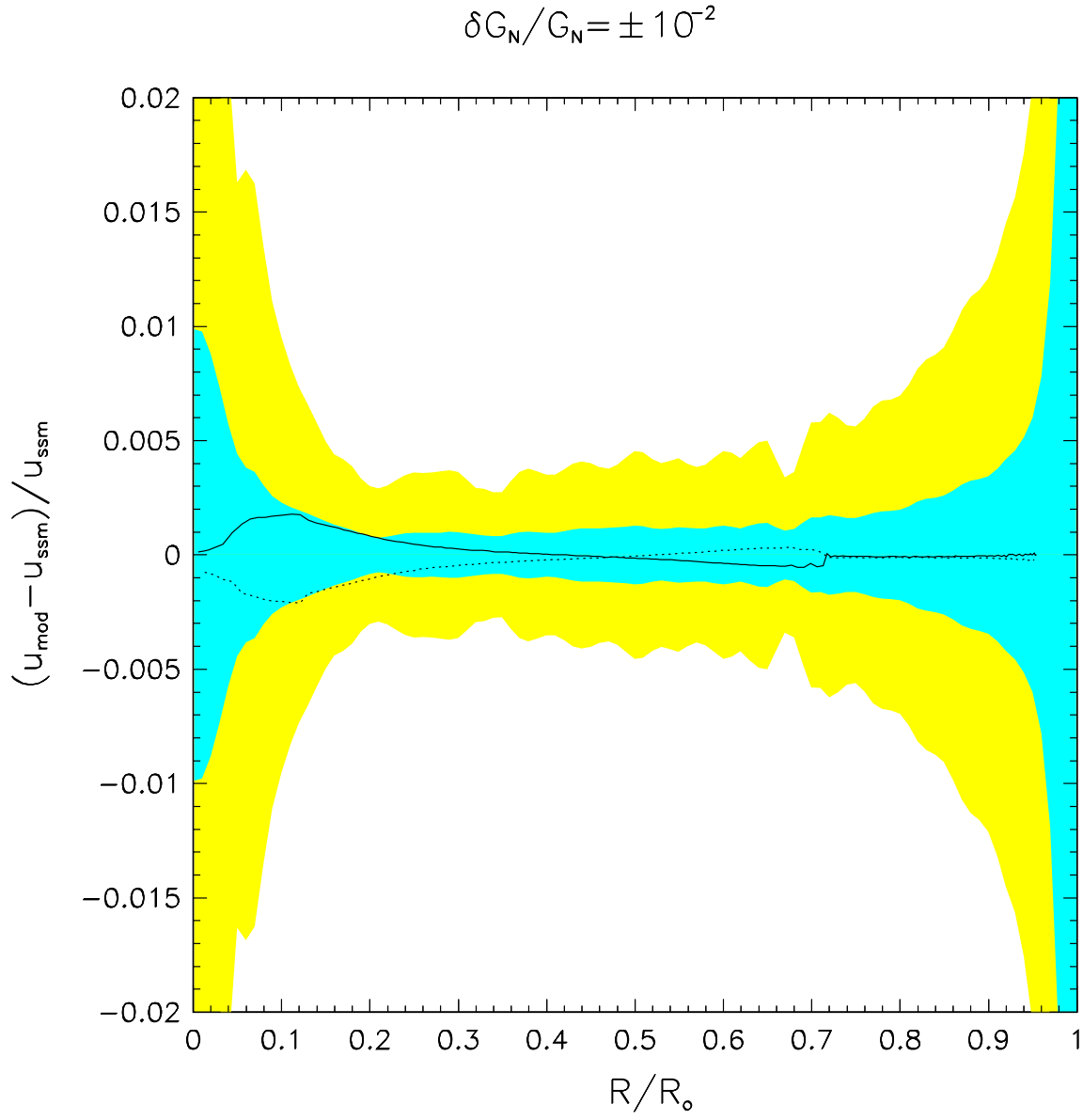


FIG. 2. Relative change (model-SSM)/SSM of $u = P/\rho$ as a function of the radial coordinate, for a change of G_N by +1% (full line) and -1% (dotted line). The 1σ (3σ) helioseismic uncertainty correspond to the dark (light) area.

$$\delta G_N/G_N = \pm 10^{-2}$$

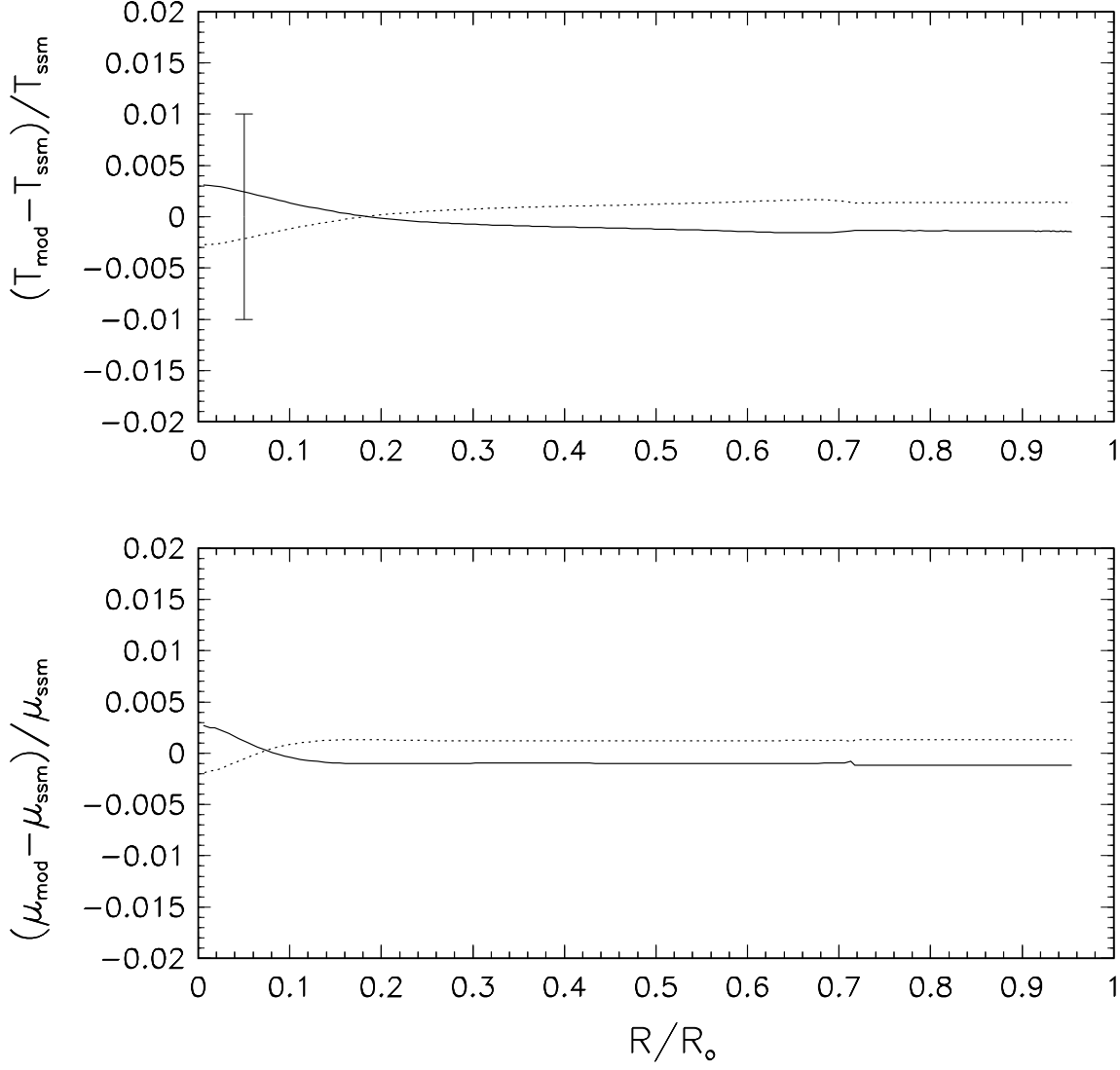


FIG. 3. Relative change (model-SSM)/SSM of temperature (upper panel) and mean molecular weight (lower panel) as a function of the radial coordinate, for a change of G_N by +1% (full line) and -1% (dashed line). The vertical bar in the upper panel corresponds to the 1σ uncertainty on T_c from neutrino measurement.

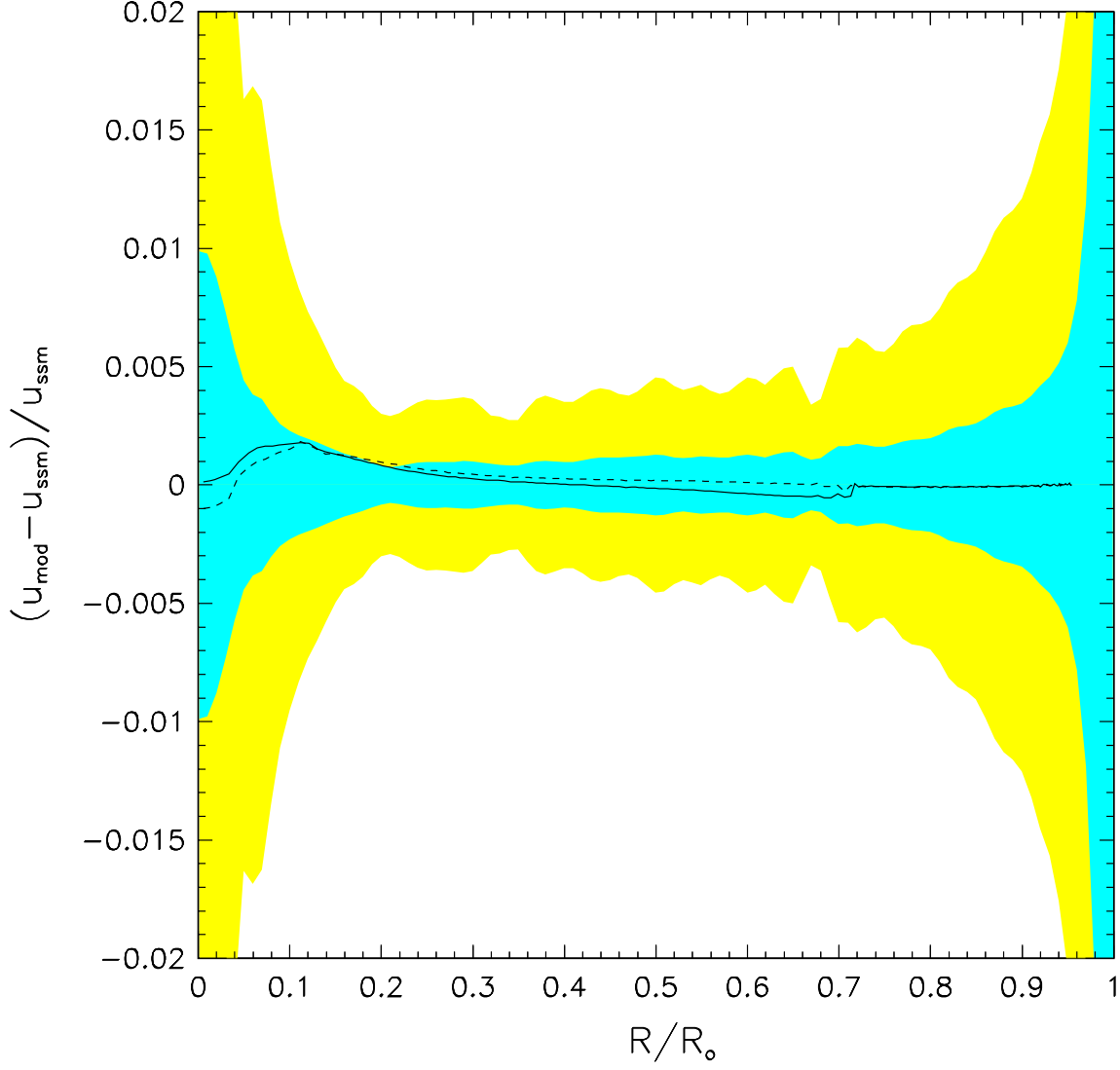


FIG. 4. Relative change (model-SSM)/SSM of $u = P/\rho$ as a function of the radial coordinate, for a change of G_N by +1% (full line) and for a solar model with energy production rate decreased by 2% and opacity decreased by 1% (dashed line). The 1σ (3σ) helioseismic uncertainty correspond to the dark (light) area.